Some Mathematical Considerations in the Predictability of Weather

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LONG-TERM GOALS

My long term goal is to understand the fundamental limitations on short-term, mesoscale predictability. Of particular interest are the effects of mid-scale turbulence on episodic initiations, or triggering mechanisms, and the sensitivities of the predictions of consequential events to these mechanisms. The issue is to develop techniques by which we can objectively distinguish between hard and impossible prediction problems. We are especially interested in applying these results to provide guidance for the design and interpretation of the ensemble forecast systems.

OBJECTIVES

I wish to investigate the concept of a well-posed prediction problem, a prediction problem that has a theoretical solution. The complementary set of ill-posed prediction problems are those for whom no prediction algorithm or model can be developed, which will satisfy the required forecast accuracy or skill. A prediction problem may be well-pose, even though the prediction technology has not yet been developed. Thus the well-posed concept separates the impossible predition problems from those that are just very challanging. Studying well-posed prediction problems includes quantifying the relationship between the prediction goals and the verification results.

These investigations also involve considerations of the methodology for quantifying the skill of prediction systems. Since the specification of a well-posed problem includes the selection of an objective verification procedure, it is necessary to have a sound statistical foundation for verification. We examine how this paradigm applies to a simple classical chaotic system and predictions of states on its attractor. Studies of the predictability of the Logistic Mapping from various initial states on its attractor provide insight. The notion of condition entropy is introduced as the skill metric for these studies.

APPROACH

Traditional statistical measures of predictability of chaotic systems, such as its invariant measure and entropy, measure the climatology of the attractor. Conditional measures relate to predictability for specific initial states. We develop techniques for computing the conditional probabilities and the associated conditional entropy. The conditional entropy estimates the number of lost bits of information between the initial condition and the prediction. Thus if a forecast from the initial

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condition x is initiated with 8-bit precision, and the forecast has conditional entropy equal to 2, then the prediction is expected to have 6-bit accuracy.

We also investigate the statistical estimation of skill metrics for categorical forecasts. These metrics are candidates for measuring the skill of traditional weather forecast systems, and sample estimations are useful only if they are repeatable, that is, they do not depend on weather frequency during a verification exercise. In the course of these studies, we also determine the relationship of skill statistics to ecconomic benefits.

WORK COMPLETED

We have completed two studies this year:

1. An extensive study of skill statistics for categorical forecast systems has been completed. Both deterministic and probabilistic forecasts are investigated.
2. The notion of conditional entropy has been developed, together with an estimation procedure. This measure has been applied to several cases of the Logistic Mapping and it has been found that there are cases where the traditional predictability measures are unduly pessimistic.

RESULTS

Skill statistics for deterministic binary forecasts are investigated from the viewpoints of repeatability, and relevance. These statistics are discussed from the viewpoints of conditional probabilities, their estimation, and their relationships to Type I and Type II forecast skill. Most of the classic skill statistics for deterministic forecasts are relevant, but the TSS is the only repeatable classic skill statistic.

The skill statistics are interpreted in terms of the expected benefits of applications; two variants of the TSS are found to be important.

The worth skill statistic WSS_U is formed by rescaling TSS, based on the application use factor U, which is defined in terms of the user's gain-loss factors. WSS_U is proportional to the benefits recovered and to the benefits-per-trial for the class of users with use factor U.

The worth ratio WR is the ratio of the terms of TSS. Tuning a forecast system to maximize WR will produce nearly maximum values for each WSS_U, and will provide maximum benefits-per-action for all users. WR is a valuational skill statistic; whenever there is a change to a forecast system that increases WR, then there is a corresponding increase in benefits-per-action for all users. The WR & WSS Graph is introduced as tool for optimizing a forecast system that depends on a continuous parameter. It is an extension of the Relative Operating Characteristic Diagram of Mason.

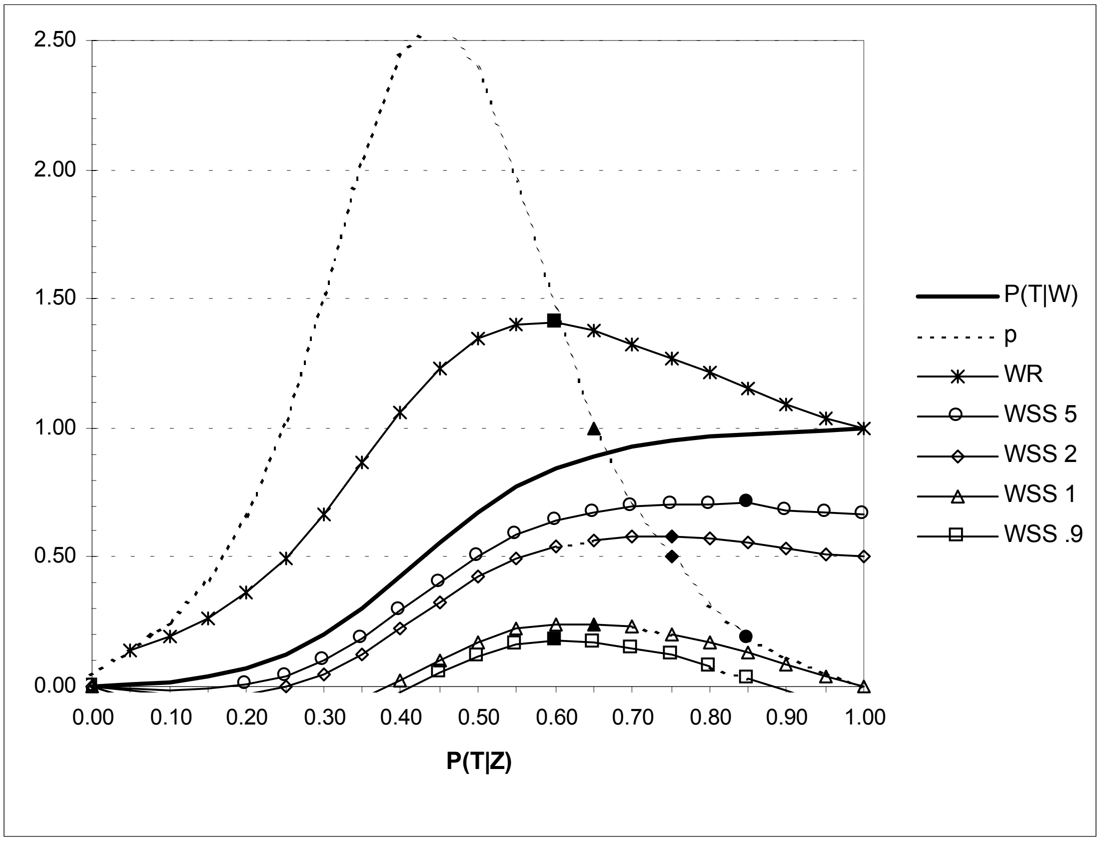

Figure 1. The WR & WSS Graph, plotted against P(T|Z) for synthetic data. The maximum values of each WSS_U curve are marked as solid, together with the corresponding point on p. Note the locations of WSS_U maxima to the right of the maximum of WR. Recall that WSS_1 = TSS. The function p represents the derivative of P(T|W) in this parameterization, and is included to illustrate the theoretical relationships.

A context is developed that provides extensions of these skill statistics to probabilistic binary forecasts. Since the extensions are defined in terms of the conditional probabilities, the notions of repeatability and relevance remain applicable. The popular Brier Score is found to be not repeatable. The extensions of TSS, WSS_U, and WR provide repeatable and relevant measures of the skill probabilistic forecasts. These measures also have the property that they have a consistent interpretation as probabilistic forecasts become more certain.

In order to determine the value of probability forecasts in applications, it is necessary to consider the action strategy of the user. Two action strategies are considered: full commitment and partial commitment. The benefit analysis for deterministic forecasts is directly applicable to the full commitment strategy and to the simple partial commitment strategy. We show that the optimal partial commitment strategy is a full commitment strategy. The WR & WSS Graph provides a graphical process for determining the decision threshold that optimizes the commitment strategy for probabilistic forecasts.

Our second class of investigations relates to studies of the predictability of chaotic systems. The analysis of these systems is conducted through numerical experiments. A key factor is the predictability, with a perfect model, when there is slight uncertainty in the initial condition. The probability distribution of the resulting forecasts is a key factor. We represent the density of this distribution by the conditional probability $p(y|x)$, where y is a possible forecast and x is the initial condition i the space X. Conditional entropy converts this condition probability to a measure of the expected loss of resolution in the forecast:

Conditional Entropy of y:	$I(y\|x) = - \log p(y\|x)$	the uncertainty in $y\|x$
Conditional Entropy of forecasts:	$I(Y\|x) = \int p(y\|x)\, I(y\|x)\, dy$	average uncertainty $\|x$
Conditional Entropy of X:	$I(Y\|X) = \iint p(x,y)\, I(y\|x)\, dydx \ = \ \int p(x)\, I(Y\|x)\, dx$	
	the average uncertainty of all forecasts on X.	

We compute the conditional entropy of the logistic map for r = 3.826, a location where the Logistic Mapping is strongly chaotic and has entropy 10.3 for 12-bit initial conditions. We see that there is a substantial variation of the predictability across the attractor, that forecasts from the initial conditions near .5 remain predictable (Figure 1), and that there is a linear degradation of the predictability with forecast lead time (Figure 2). We make no special claim about this last result, even though we have seen it repeatedly as we have varied r. It may be a property of the Logistic Mapping, rather than a predictability result.

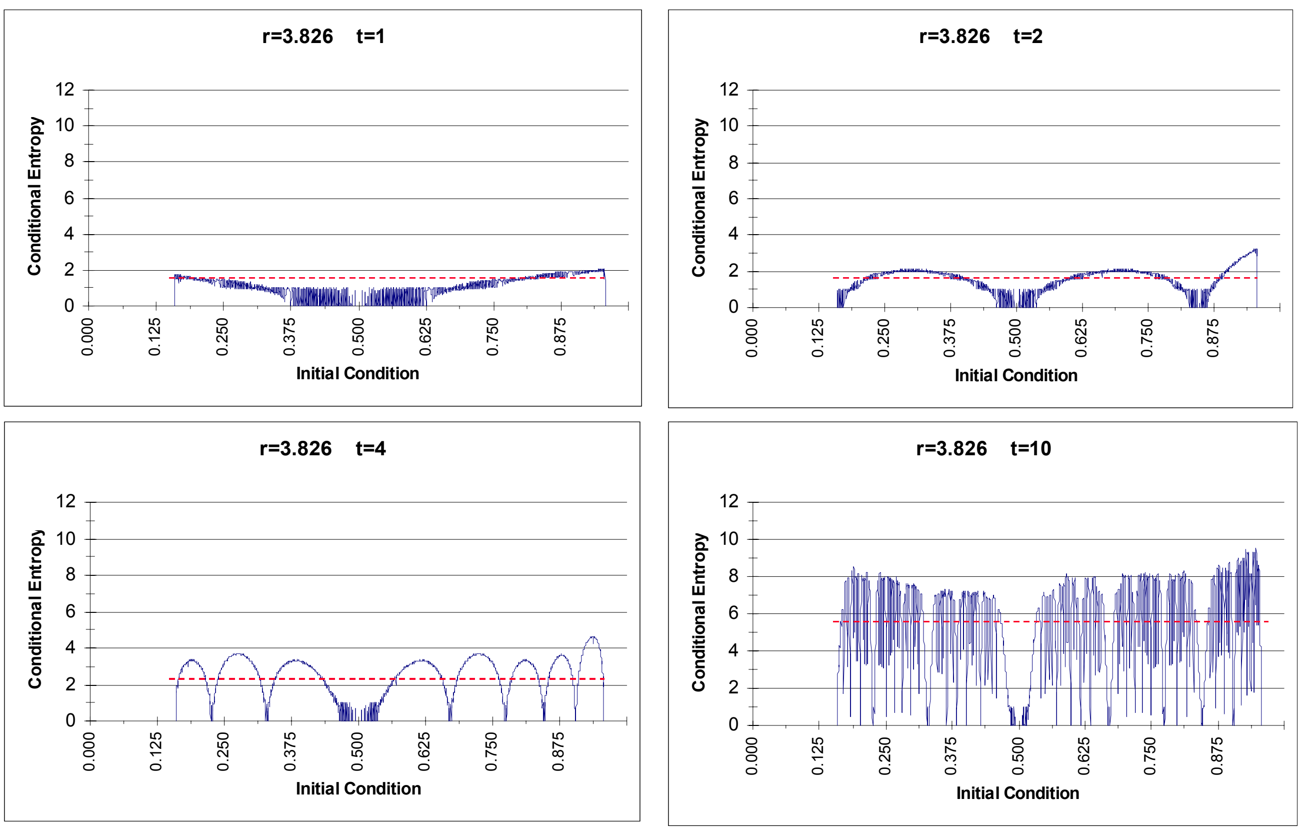

Figure 2. Conditional Entropy I(Y|x) as a function of x for perfect forecasts of time 1,2,4, and 10 from the attractor of the Logistic Mapping with r=3.826.

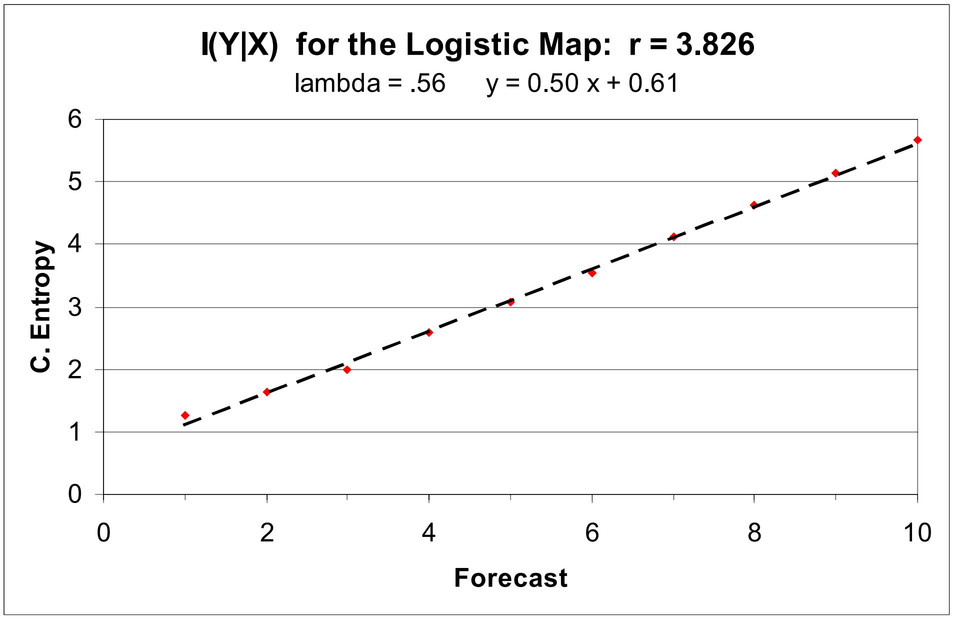

Figure 3. The change in the average conditional entropy as a function of forecast lead time for perfect forecasts of the Logistic Mapping with r=3.826. Lambda is the average Lyapunov exponent.

IMPACT/APPLICATIONS

The development of skill statistics that are dependable and which reflect user benefit has wide potential application. These measures of skill have the additional merit that they unify the scoring of deterministic and probabilistic forecasts.

TRANSITIONS

The results concerning skill statistics are being considered for the evaluation of the skill of forecast products in the FAA' s Aviation Weather Research Program.

RELATED PROJECTS

The skill statistics have been used to evaluate the forecast skill for four forecast models that were developed by the FAA' s Aviation Weather Research Program for the forecast of the dissipation of low clouds at San Francisco International Airport.

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